IMPACT OF POINTING ERRORS ON THE PERFORMANCES OF DOUBLE RICIAN FSO CHANNELS

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Contribution to the State of the Art
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Abstract: In this paper we will propose new analytically traceable probability density function (PDF) model for free space optics (FSO) turbulence, obtained as a generalization of double Ricean turbulence model, that encompasses both large-scale and small-scale turbulence eddy effects along by taking into account performance decreasing influence of misalignment introduced through boresight pointing error model. Consequently, after delivering the closed-form expressions for the newly introduced double FSO model, we obtain the analytical expressions for the bit error rate (BER) performance for the Double Rician distribution affected by misalignment. Numerical results will show the impact of system parameters on FSO link performance and we will provide full performance analysis. © 2021.

Keywords: free space optics (FSO), atmospheric turbulence, pointing error, bit error rate (BER).

INTRODUCTION

Providing the tractable probability density function (pdf) to describe atmospheric turbulence is important in order to study the performance characteristics of a free space optics (FSO) transmission system [1]-[6]. Over the last decades, a number of statistical models have been proposed for different degrees of turbulence severity. However, in addition to the scintillation effect, misalignment between transmission point and reception point due to building wobble causes oscillation of the FSO beam and, thus, pointing errors that restrict the performance of atmospheric FSO links, so the impact of pointing error (jitter) was widely investigated in literature. Recently, a new and generalized statistical model, called double Rician distribution, was used to model the the irradiance fluctuations of an unbounded optical wavefront propagating through a homogeneous, isotropic channel [7], [8], [9]. Double Rician distribution was demonstrated to have the advantage of unifying most of the proposed statistical models delivering, additionally, an excellent settlement with measured characteristics of plane wave and spherical wave propagation for a wide range of observed turbulence conditions (weak, moderate, strong). Here, such a model is upgraded by encompassing the harmful influence of pointing error losses that occur due to misalignment. In such esteem, familiar effects of aperture size, beam width and jitter variance are brought into consideration. In this work, we are introducing a novel general turbulence model, named as Double-Rician, for representing broad range of irradiance fluctuations. The parameters of the presented model pdf are connected straightly to the atmospheric parameters. Based on the Rytov approach which is commonly used in scintilation theories, we observe the synthesized turbulence as the product of smal-scale and large-scale turbulences, both analytically modeled by the Rician distribution model, and further derive closed-form expressions for their pdf and cumulative density (cdf) functions expressed through Meijer's G-function. Further, such a model is completed by by encompassing the harmful influence of pointing error losses that occur due to misalignment. In this regard, prominent upshots of aperture size, beam width and variance of jitter are taken into consideration. Consequently, after introducing the closed-form expressions for the synthesized composite distribution of scintillation and pointing errors, we obtain the closed-form expressions for the BER performance of an IM/DD for the Double-Rician distribution affected by misalignment, exposuring the impairment in performance of pointing errors in atmospheric FSO links. Numerical results show the impact of misalignment on link performance.

ATMOSPHERIC DOUBLE-RICEAN MODEL

Let us here introduce closed form of new Double-Rician model as the product of two types of fluctuations. The optical turbulence channel can be modeled as the modified Rician distribution, where both the amplitude and phase of the optical signal fluctuate. The intensity fading h_j, j = 1, 2, induced by optical atmospheric turbulence can be statistically described by:

$$f_{h1}(h_1) = \frac{1+K_1}{\overline{h_1}} \exp\left(-K_1 - \frac{(1+K_1)h_1}{\overline{h_1}}\right) \times I_0\left(2\sqrt{\frac{h_1(1+K_1)K_1}{\overline{h_1}}}\right) \qquad (1)$$

$$f_{h2}(h_2) = \frac{1+K_2}{\overline{h_2}} \exp\left(-K_2 - \frac{(1+K_2)h_2}{\overline{h_2}}\right) \times I_0\left(2\sqrt{\frac{h_2(1+K_2)K_2}{\overline{h_2}}}\right) \qquad (2)$$

where $\overline{h}_{\mathrm{j}}$, j = 1, 2, is the mean value of the turbulence intensity, K_j is reciprocal of the contrast parameter and $I_0(x)$ is the firstkind modified Bessel function of order zero. $\overline{hj}$ and K_j can be expressed as [10],[11]:

$$K_j = \left(\frac{\overline{\alpha}_{rj}^2+\sigma_{rj}^2+\sigma_{ij}^2}{\sqrt{\overline{\alpha}_{rj}^4+2\overline{\alpha}_{rj}^2\left(\sigma_{ij}^2-\sigma_{rj}^2\right)-\left(\sigma_{ij}^2-\sigma_{rj}^2\right)^2}} - 1\right)^{-1}, \quad (3)$$

$$\overline{h_j = \overline{\alpha}_{rj}^2} + \sigma_{rj}^2 + \sigma_{ij}^2, \quad j = 1,2; \qquad (4)$$

where $\overline{\alpha}_{rj}$, denote the mean values of real part of the effective turbulences coefficients, while σ_{rj}^2 and σ_{ij}^2 denote the variances of the real and imaginary parts of the effective turbulences coefficients, respectively:

$$\sigma_{rj}^2 = \frac{1+\exp\left(-2\sigma_{\varphi j}^2\right) - 2\exp\left(-\sigma_{xj}^2 - \sigma_{\varphi j}^2\right)}{2N}$$

$$\overline{\alpha}_{rj} = exp\left(-\frac{\sigma_{xj}^2+\sigma_{\varphi j}^2}{2}\right)$$

$$\sigma_{ij}^2 = \frac{1-\exp\left(-2\sigma_{\varphi j}^2\right)}{2N}, \qquad j = 1,2; \qquad (5)$$

with $\sigma_{\varphi j}^2$ and σ_{xj}^2 denoting the variances of amplitude fluctuations and phase variations respectively, while N representing the number of statistically independent cells in the receiver aperture (RA), defined as:

$$N = \left(1.09(r_0/D)^2\gamma\left(6/5, 1.08\,(D/r_0)^{\frac{5}{3}}\right)\right)^{-1}, \qquad (6)$$

where D denotes the aperture diameter, r_0 denotes the wavefront coherence diameter and $\gamma(a,x)$ stands for the lower incomplete Gamma function. The log-normal amplitude variances $\sigma_{\varphi j}^2$ are related to a scintillation index $(SI), \sigma_{\beta j}^2 = exp(4\sigma_{\varphi j}^2) - 1$, while the Gaussian phase variances σ_{xj}^2 are defined as $\sigma_{xj}^2 = C_J \left(\frac{D}{r0}\right)^{5/3}$, where C_J is determined by the number (J) of Zernike terms corrected by active modal compensation. As a result, the intensity fading of the optical signals over the modified Rician channels are determined by both the amplitude scintillation and phase aberration.

According to Rytov principle [2], [5] the resulting irradiance of the received optical wave can be represented through a product of two types of fluctuations, i.e. $h_a - h_1 h_2$, appearing from both large and small scale turbulence eddies, respectively. It is supposed that h_1, h_2 are mutually independent random processes and, hence, the second moment of the irradiance is expressed as :

$$E\langle h_a^2 \rangle = \langle h_1^2 \rangle \langle h_2^2 \rangle = \left(1 + \sigma_{h1}^2\right)\left(1 + \sigma_{h2}^2\right), \qquad (7)$$

where σ_{h1}^2 and σ_{h2}^2 are normalized variances of $h1$ and $h2$, respectively. Without loss of generality, we suppose $E\langle h_a \rangle = 1$ or analogously $E\langle h_1 \rangle = 1$ and $E\langle h_2 \rangle = 1$. Then the scintillation index can be expressed as:

$$\sigma_{ha}^2 = \frac{E\langle h_a^2 \rangle}{E\langle h_a \rangle^2} - 1 = \left(1 + \sigma_{h1}^2\right)\left(1 + \sigma_{h2}^2\right) - 1. \qquad (8)$$

Let us suppose that both type of scale of the irradiance fluctuations are conducted by the Rician distribution, i.e. h_1, h_2 and poses the pdfs of (1) and (2) with parameters K_1, $\overline{h_1}$ and K_2, $\overline{h_2}$, respectively. Then, based on the method proposed in [5], the pdf of h_a can be written as:

$$f_{h_a}(h_a) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \frac{2(1+K_1)^{\frac{p+q+2}{2}}(1+K_2)^{\frac{p+q+2}{2}} K_1^p K_2^q}{\overline{h_1}^{\frac{p+q+2}{2}} \overline{h_2}^{\frac{p+q+2}{2}} \Gamma(p+1)\Gamma(q+1)p!q!} h_a^{\frac{p+q}{2}}$$
$$\exp(-K_1) \exp(-K_2) K_{p-q}\left(2\sqrt{\frac{(1+K_1)(1+K_2)h_a}{\overline{h_1 h_2}}}\right) \qquad (9)$$

where $K_v(x)$ is v^{th}-order modified Bessel function of the second kind [12, Eq. 8.432].

MISALIGNMENT FADING MODEL

The pointing errors arise as a result of the misalignment between the transmission point and reception point. Weak earthquakes, thermal expansion and powerful wind cause buildings to oscillate which results in the vibrations of the optical beam. In order to describe the effects of pointing errors, we start from the model proposed in [13]. This model assumes a Gaussian spatial intensity profile of beam waist w_z on the receiver plane and a circular aperture of radius a. Both vertical and horizontal oscillations are modeled by mutually independent identical Gaussian distributions, so the radial displacement r at the receiver detector can be expressed by Rayleigh distribution with the jitter variance σ_s^2. The PDF of h_p can be expressed as [13], [14]:

$$f_{hp}\left(h_p\right) = \frac{g^2}{A_0^{g^2}} h_p^{g^2-1}, 0 \le h_p \le A_0, \qquad (10)$$

where $g = \omega_{zeq}/2\sigma_s$ is the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver and $\omega_{zeq}^2 = \frac{\omega_z^2\sqrt{\pi}\,\mathrm{erf}(\upsilon)}{2\upsilon\exp(-\upsilon^2)}, \upsilon = \sqrt{\pi}a/2\sqrt{\omega_z}, A_0 = [\mathrm{erf}(\upsilon)]^2$, where $erf(\,)$ is the error function [12], (8.250.1).

COMPOSITE CHANNEL MODEL STATISTICS

Here, we deliver a composite statistical model of a stochastic FSO channel by taking into account both scintillation inducted by turbulence and pointing errors inducted by misalignment. Therefore, the unconditional pdf, $f_h\,(h)$, for the channel state, h, is obtained by calculating the product of the two pdfs presented above in [14]:

$$f_h(h) = \int f_{h|h_a}(h|h_a) f_{h_a}(h_a)\,dh_a \tag{11}$$

where $f_{(h|ha)}(h|h_a)$ is the conditional probability given a turbulence state, h_a, and it is expressed as:

$$f_{h|h_a}\,(h|h_a) = \frac{1}{hp}\,fh_p\left(\frac{h}{hp}\right), \quad 0 < h \le A_0 h_a \tag{12}$$

Substituting expression (12) in (13) we get the expression for PDF in the form of:

$$f_h(h) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\frac{(1+K_1)(1+K_2)K_1^pK_2^q\exp(-K_1)\exp(-K_2)}{\bar{h}_1\,\bar{h}_2A_0\Gamma(p+1)\Gamma(q+1)p!q!}$$
$$g^2G_{1,3}^{3,0}\left(\frac{(1+K_1)(1+K_2)}{\bar{h}_1\,\bar{h}_2A_0}h\middle|\begin{matrix}g^2\\g^2-1,p,q\end{matrix}\right) \tag{13}$$

where $G_{1,3}^{2,1}(x)$ denotes Meijer G function, given with [15, Eq. (07.34.02.0001.01)], Eq. (07.34.21.0085.01), [15, Eq. (07.34.17.0011.01)].

AVERAGE BER

In this section, we capitalize on presented analysis by delivering the procedure for obtaining the closed-form expressions for the BER of the Double Rice pdf in the presence of pointing errors, when IM/DD link using OOK signaling has been considered. Mainly in practice, signal-to-noise ratio (SNR) at the reception is limited by shot noise, that arises by background light and could be much stronger than the power of the desired signal. Similarly, it can be limited by thermal noise that arises from electronic components that follow the photodetector. Because of that, noise is usually modeled to high accuracy as Additive white Gaussian noise (AWGN) that is statistically independent of the desired signal. Presence of AWGN, may lead to the occurrence of reception errors when determining the values of actual transmitted symbols is made. The overall probability of error, $P_b(e)$ can be determined by calculating following weighted sum:

$$P_b(e) = p_0p(e|0) + p_1p(e|1), \tag{14}$$

where $p_0$denotes the transmission probability of a binary symbol “0” and p_1 denotes the transmission probability of a binary symbol “1”, as pointed out in [16]. On another note, $p(e|0)$ and $p(e|1)$ denote the conditional bit error probabilities when the transmitted bit is “0” and when the transmitted bit is“1”, respectively. It is assumed that either symbol (binary “0” or binary “1”) are equally possible to be transmitted, i.e., $p_0 = p_1 = 1/2$. Assuming that $p(e|0) = p(e|1)$, it is easy to show that the conditional BER can be expressed as:

$$P_b(e|h)p(e|0,h) = p(e|1,h) = Q\left(\frac{PR_h}{\sigma_N}\right), \tag{15}$$

where P denotes the average power of optical signal, while R denoting the responsivity and σ_N^2 standing for the AWGN variance, while $Q(x)$ represents the Gaussian Q function. ABER. Now, $P_b(e)$, can be obtained by averaging $P_b(e|h)$ over $f_h(h)$:

$$P_b(e) = \int_0^\infty P_b(e|h) f_h(h) dh. \tag{16}$$

After substituting (13) into (16), and by using [15, Eq. (07.34.21.0013.01)], the ABER is derived in a closed-form expression as:

$$P_b(e) = \sum_{p=0}^{\infty}\sum_{q=0}^{\infty} \frac{2^{p+q-2} K_1^p K_2^q \exp(-K_1)\exp(-K_2)\, A_0^2}{\pi\sqrt{\pi}\Gamma(p+1)\Gamma(q+1)p!\, q!}$$
$$\times G_{2,6}^{7,4}\left(\frac{8PRh_1^{-2}h_2^{-2}A_0^2}{(1+K_1)^2(1+K_2)^2\sigma_N}\middle|\begin{matrix}\frac{1-g^2}{2},\frac{-g^2}{2},\frac{-p}{2},\frac{1-p}{2},\frac{-q}{2},\frac{1-q}{2},1\\ 0,\frac{1}{2},\frac{-g^2}{2},\frac{1-g^2}{2}\end{matrix}\right), \tag{17}$$

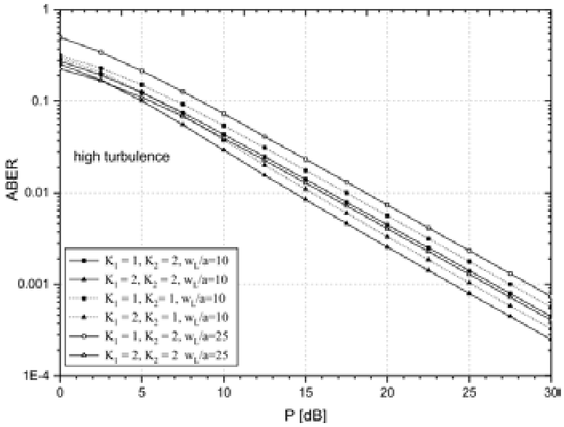

Fig. 1. ABER for the double Rician pointing error channel model for different values of the normalized optical beam radius

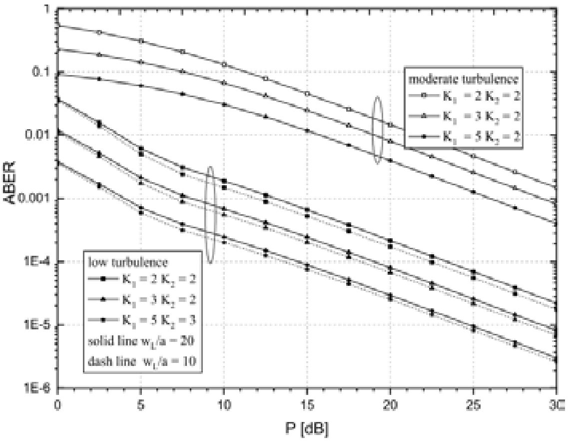

Fig. 2. ABER for the double Rician pointing error channel model for different values of the atmospheric conditions

NUMERICAL RESULTS AND DISCUSSIONS

For the numerical calculation, the FSO system was observed at the wavelength $\lambda = 1550\ nm$ and at the distance between transmitter and receiver $L = 1\ km$. Three types of atmospheric turbulence were considered: weak, moderate and strong, with indexes of reflection $C_n = 6 * 10^{-15} m^{-2/3}$, $C_n = 2 * 10^{-14} m^{-2/3}$, and $C_n = 1.2 * 10^{-13}\ m^{-2/3}$, respectively. Total received signal power is $P = 1$, while detector responsivity $R = 1\ [A/W]$ and noise variance is $\sigma_N = 10^{-7} [A/Hz]$. The radius of a circular detector aperture $a = 0.05\ m$, optical beam radius at distance L from transmitter $\omega_L = 0.5\ m$, pointing error displacement standard deviation (jitter) at the receiver $\sigma_s = 0.2m$ are considered.

In Fig. 1 and Fig. 2 are shown ABER behavior of double Ricean channel model, with applied OOK modulation scheme as a function of the average optical power at the transmission P, for different values of the normalized optical beam radius ω_L at distance L from transmitter. Since optical beam radius ω_L depends on the distance L from the transmitter, it is clear that higher values of this parameter greatly affect the deterioration of ABER, as well as the system performance. As expected, for stronger turbulence and lower values of the K factor, the higher ABER values are obtained. In both the atmospheric turbulence and the pointing error model, the ABER decreases significantly faster for higher values of the K factor then for those lower ones.

CONCLUSIONS

Double Rician scintillation channel model was demonstrated to have the asset of encompassing most of the proposed channel models derived until now in the literature in a closed-form expression, while delivering, additionally, an excellent settlement with measured characteristics of plane wave and spherical wave propagation for a wide range of observed turbulence conditions (weak, moderate, strong). Here, this model has been upgrading by incorporating the harmful effects of pointing error losses due to misalignment. In this regard, prominent effects of aperture size, beam width and variance of jitter are taken into consideration. In this work we have presented the analytical closed form representations for the PDF of the irradiance at the receiver, as well as of instantaneous SNR for the newly introduced double Rician model in the presence of atmospheric turbulence and pointing error are derived. Finally, we obtain closed-form expressions for the BER characteristics of an IM/DD for the Double-Rician distribution affected by misalignment.

REFERENCES

[1] Antonio Jurado-Navas, José María Garrido-Balsells, José Francisco Paris, Miguel Castillo-Vázquez, and Antonio Puerta-Notario, “Impact of pointing errors on the performance of generalized atmospheric optical channels,” Opt. Express 20, 12550-12562 (2012)

[2] L.C. Andrews, R.L. Phillips Laser Beam Propagation through Random Media (2nd ed.), SPIE Press, Washington, USA (2005)

[3] N.D. Chatzidiamantis, H.G. Sandalidis, G.K. Karagiannidis, S.A. Kotsopoulos, M. Matthaiou, New results on turbulence modeling for free-space optical systems, in: Proceedings of IEEE International Conference on Telecommunications, 2010, 487–492.

[4] J.A. Jurado-Navas, J.M. Garrido-Balsells, J.F. Paris, A. Puerta-Notario, A unifying statistical model for atmospheric optical scintillation, arXiv preprint arXiv:1102.1915, 2011.

[5] M. Al-Habash, L.C. Andrews, R.L. Phillips, Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media, Opt. Eng., 40 (2001), pp. 1554-1562.

[6] G. Parry, P.N. Pusey, K distributions in atmospheric propagation of laser light, J. Opt. Soc. Am., 69 (1979), pp. 796-798.

[7] Panić S., Milošević H., Prlinčević B., Performance analysis of FSO transmission of double watermarked image over the Double Rician turbulence channel, Proceedings of First West Asian Colloquium on Optical Wireless Communications (WACOWC), April 25, 2018, Isfehan, Iran.

[8] Panić S., Milošević H., Vasić S, Milenković V., Dynamical characteristics of the FSO transmission capacity in the presence of Rician turbulence, Proceedings of 2018 International Conference on Information and Communications Technology (ICOIACT), March 6-7, 2018, Jogyakarta, Indonesia.

[9] Prlinčević B. P., Panić S. R., Spalević P. C., Mišić M. A., Amniesi A., Stanojević V., On the transmission of double watermarked image over Rician FSO channel, Eletronika ir Elektrotechnika 22(3), 2016, pp. 83–88, DOI: 10.5755/

j01.eie.22.3.15320.

[10] A. Belmonte, J. M. Kahn, "Performance of synchronous optical receivers using atmospheric compensation techniques", Opt. Exp., vol. 16, no. 18, pp. 14151-14162, Sep. 2008.

[11] C. Chen, W-D. Zhong, X. Li, D. Wu, "MDPSK-based nonequalization OFDM for coherent free-space optical communication", IEEE Photon. Technol. Lett., vol. 26, no. 16, pp. 1617-1620, Aug. 2014.

[12] Gradshteyn I. S., Ryzhik I. M., Table of integrals, Series, and Products, 7th Edition, Elsevier Academic Press, 2007.

[13] A. A. Farid and S. Hranilovic, "Outage capacity optimization for free-space optical links with pointing errors," J. Lightwave Technol. 25, 1702–1710 (2007).

[14] Smilić M., Perić Z., Milić D., Marković A., Savić M., Influence of zero and non-zero boresight pointing errors on biterror rate free-space optic transmission over Málaga atmospheric channel, Optica Applicata 48(2), 2018, pp. 179–189, DOI: 10.5277/oa180202.

[15] http://functions.wolfram.com/

[16] Tsiftsis, G. K. Karagiannidis, and M. Uysal, "BER performance of FSO links over strong atmospheric turbulence channels with pointing errors," IEEE Commun. Lett. 12, 44–46 (2008)

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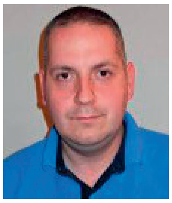

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